Faculty of Engineering
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CANTILEVER TRUSS DESIGN PROJECT

Professor: Kamyar Ghavam
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1 EXECUTIVE SUMMARY

The purpose of this project was to apply knowledge gained through various solid mechanics courses to a real-life design project. The project provides real life application of numerous in-class concepts including force and failure analysis, design and resource constraints, and truss construction. This project also carries a heavy significance on design optimization, with a primary goal being to achieve the optimal strength to mass ratio.

The key idea used in the optimization process was to tend the optimization of the dimensions towards infinity. This idea was used because of the balsa wood being used as the truss members. The natural wood cracks would be more problematic if the dimensions were smaller, as this would increase crack density in the members. However, if instead the dimensions are optimized towards infinity, then these cracks become less significant and the truss can support more load. When it came to geometric design, the base design is of a simple single triangle, with box beams as the horizontal compressive members, and slender stacked tensile members. The specific geometry of these members was shifted around during the optimization of later designs, but maintained the key concepts.

Results of initial designs and tests showed mainly issues with size. The concept to optimize to infinity resulted in designs that were too large for the testing rig, so designs needed to be modified specifically to follow this methodology but fit within the rig. Overall this methodology was well utilized. The truss can perform well under load, and only had some dimensions changes required to optimize force distribution.

2 INTRODUCTION

2.1 Design Problem

The design problem addressed in this project is to design and construct the optimum crane boom truss. The truss members are constructed by laser cutting balsa wood, with wooden dowels used for pins.

The truss should be able to support loads applied at the end of its arm, and is judged by the ratio of the max applied weight vs. the truss' weight.

2.2 Design Constraints

The design must be constructed from the given materials, however treating the balsa wood is allowed. The truss must be secured at one end with the load applied at the free end. The members must be pin-connected two-force members (excluding the load application member), and be able to freely rotate about the pins. The dimensions are a 30cm long truss arm that is 5cm tall and 6cm wide. An example given in the project manual is shown below in Figure 2.1.

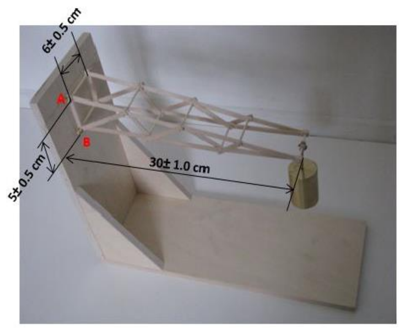


Figure 2.1 - Truss Loading Example

2.3 Design Criteria

The key design criteria are to obtain the largest possible PV value before failure. The formula for PV is shown below:

$$Performance\ Value\ (PV) = \frac{Maxmimum\ Applicable\ Load\ (g)}{Mass\ of\ Truss(g)}$$

A minimum of 75 is required, and every load must hold for at least 5 seconds to count. Accounting for the largest PV value includes accounting for various failure modes in the pins & members, as well as optimizing the weight of the truss through member design & dimensions.

2.4 Material Properties

The material properties used in calculations (excluding shear stress) for the members were determined during a three-point being experiment on balsa wood samples. Each group gathered their own data, and was then compiled together and given back to use for calculations. The values for the dowels were given in the project manual. This information is shown below in Table 2.1.

	Dowels	Members
Density ρ (g/cm^3)	0.65	0.128
Ultimate Strength σ_{ult} (MPa)	117	7.157
Young's Modulus E (GPa)	17	2.539
Shear Strength τ_{ult} (MPa)	23	2
Max Bending Moment (Nm)	0.368	N/A

Table 2.1 – Material Properties Data

The data was compiled & analyzed and then compared to data from the same experiment from previous years. Overall the rupture force values gathered were much lower than before, perhaps showing weaker wood samples used during testing. Possible sources of error include not reading the dial & applying loads in the same consistent between trials, leading to possibly inconsistent results.

3 PRELIMINARY DESIGN

During the initial design phase of the truss, many different ideas were considered. The distinct designs that the group came up with were analysed in depth, and several iterations were produced to come up with the most optimised model. This section will discuss some of the main features and designs that were considered, and then go more in-depth regarding the final chosen design.

The first design the group considered attempted to replicate structures commonly seen in structures such as cranes and bridges, as can be seen in Figure 3.1.

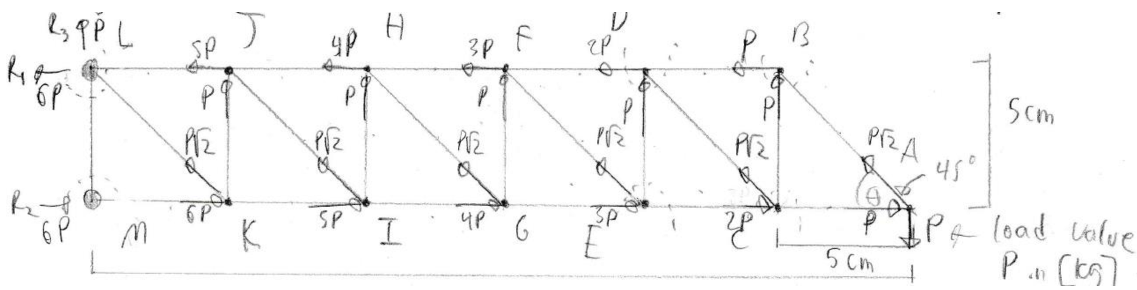


Figure 3.1 - Initial Design with Force Analysis

This design proved to be relatively effective at first, yet upon running further calculations it proved to be nearly impossible to achieve a PV of over 125; the cross-sectional area of the members would have had to be incredibly small, which would have been very hard to manufacture given the tolerances of the laser cutter used.

The second design, as shown below in Figure 3.2, we considered tried to make use of the geometrical advantages of arcs, and it employed a Warren truss. This design was further analysed, and the group concluded that the compressive members would likely buckle when the load was applied, which would compromise the integrity of the truss.

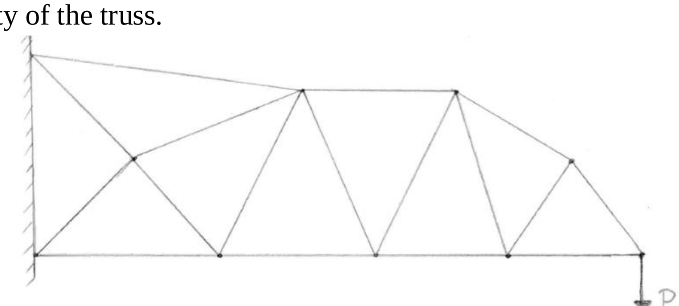


Figure 3.2 - Arc Shaped Warren Truss Design

The group then considered that the moment of inertia of the compressive members should be increased, and the use of I-beams was proposed. However, given that the balsa wood we were given had a high standard deviation—which would yield unpredictable results—the group decided that a box beam was likely to have a better performance than a makeshift I-beam.

Upon brainstorming, it was proposed that reducing the number of members and making stronger and longer members could potentially yield good results. Therefore, a simple triangular cantilevered truss was considered, and it proved to be effective when calculations were run. This design—composed of long tensile members that attach to a dowel at the end of the truss, and two long box-beams to hold the compressive load—went through further iterations. The initial design is shown below in Figure 3.3.

Making a single wide box beam to hold all the compressive load was considered, but the legality of

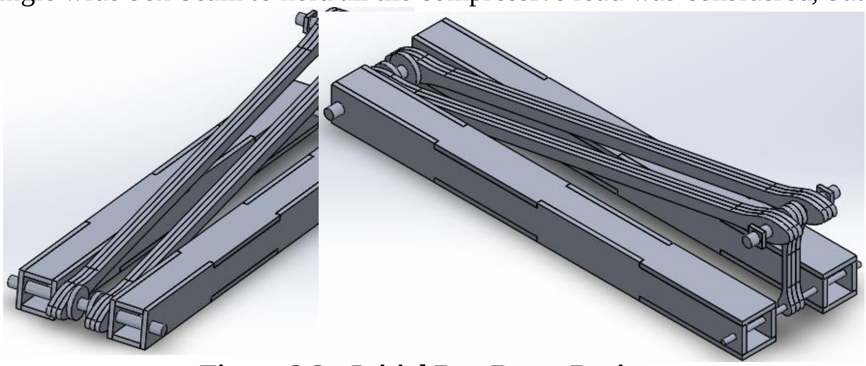


Figure 3.3 - Initial Box Beam Designs

the design was questioned (this is later addressed in section 5.1). The group decided to reinforce the box-beams by making them solid on the inside, further increasing their moments of inertia. The tensile members also went through further iterations, and "bulbs" were made around the pin holes to increase the area and thus make them less prone to tear out failure. This design is shown below in Figure 3.4.

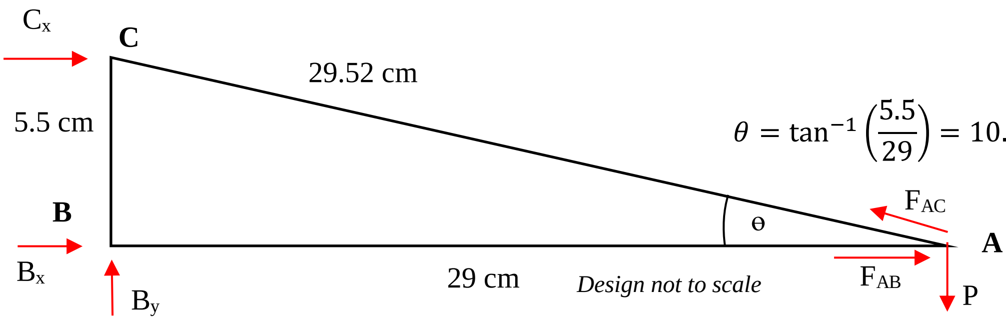
This is the final design the group decided to go with, given that it had the greatest strength to weight

Figure 3.4 - Final Geometric Design

ratio (theoretically). It was also the most scalable design, meaning that certain components could be modified after testing it. Additionally, it was designed in such a way that it would not be hard to assemble—e.g. the different parts that make up the box-beam all fit together, and the glue simply holds them in place—which would reduce the time needed to test and iterate on the design. The simplistic design also meant that the weight of the truss would be relatively light, which would improve the PV value.

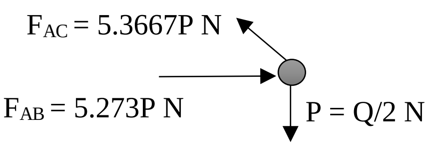
4 DESIGN ANALYSIS & OPTIMIZATION

4.1 Force Analysis



$$\theta = \tan^{-1}\left(\frac{5.5}{29}\right) = 10.73889°$$

Force Analysis: A



Moment About B:

$$\sum M_B = -C_x(0.055) - P\,(0.29) = 0$$

$$C_x = \frac{58}{11}P = 5.273P\ N$$

Moment About C:

$$\sum M_C = B_x(0.055) - P\,(0.29) = 0$$

$$B_x = \frac{58}{11}P = 5.273P\ N,\ \ B_y = 0\ N$$

As the truss has a parallel design, the relative force for each member is shown in terms of P, which is half of the applied load Q. By setting Q = max shear force before pin failure which is determined below:

$$Q = \tau_{ult} * Area = (23\ MPa) * \left(\frac{\pi * (3.175mm)^2}{4}\right) = 182.098\ N$$

$$P = \frac{Q}{2} = 91.05\ N$$

$$F_{AC} = 5.3667P = 488.64\ N$$

$$F_{AB} = B_x = C_x = 5.273P = 480.076\ N$$

4.2 Failure Analysis

Due to our choice to optimize towards infinity, we try to take the largest load using as little balsa as possible. This was done primarily because wood is a porous material, which is susceptible to crack growth and propagation. Additionally, the inherent cracks and imperfections that can be found in the grain relative to the members would be smaller. It is also notable that manufacturing imperfections will weaken large members significantly less than small ones (a 3mm long cut along a 5mm long member is more substantial than on a 20mm long member).

As all our members are being designed and constructed by us, these components can be made to meet any load (within the limits that are intrinsic to the balsa itself). Therefore, we can conclude that our ceiling will come from the hardwood dowels.

For single dowel pins, the force required for shear failure is shown below:

$$\frac{\tau_{max} * \pi * d^2}{4} = (23 * 10^6)(0.003175)^2 \times (\pi) \div 4 = 182.0980003N$$

Pin failure due to bending moment would be at the load pin as it is under three point bending while the support pin is under four point bending. The maximum applicable load for bending moment, P in newtons can be found using the equation below

$$368N \cdot mm = \frac{P(L)}{4}, Where\ L\ is\ the\ unsupported\ length\ in\ mm$$

As we wanted to increase the size of the members (optimize to infinity) and take the largest load possible, we first considered the constructing our own pins from multiple dowels. The calculations for this idea are shown below in Figure 4.1.

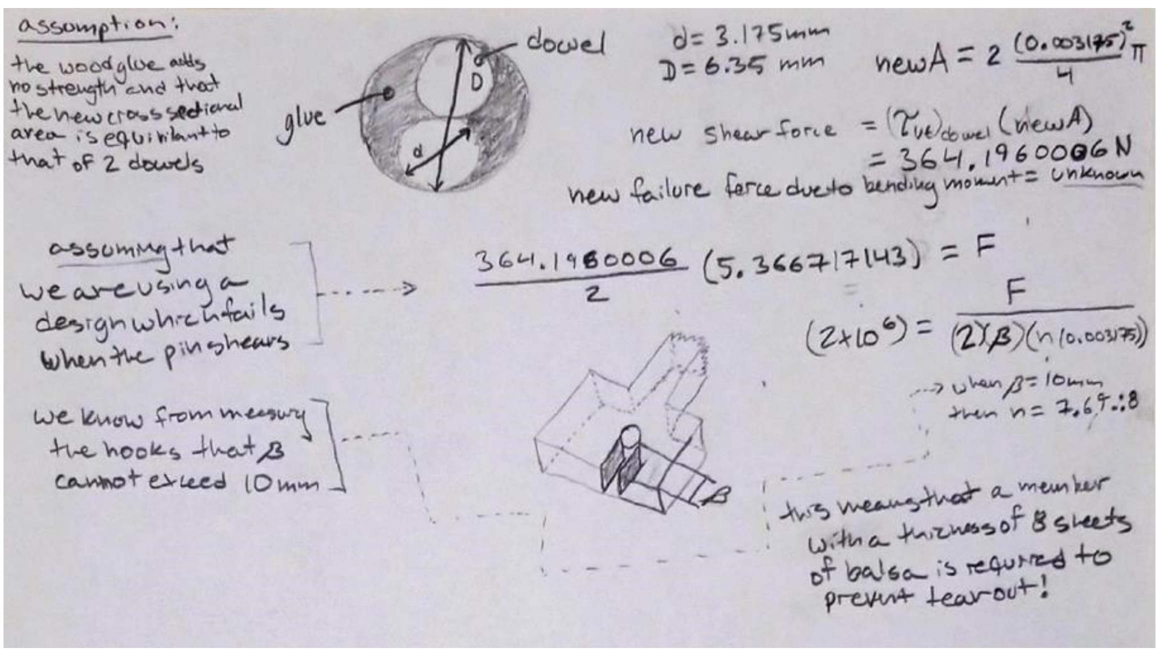


Figure 4.1 – Multiple Dowel Pins

Many assumptions had to be made and the max bending moment of the new dowel was still unknown. To add to this, we would be subtracting 16 times the thickness of the balsa from our width constrain leaving little room for compressive members. At the max load taken from our double dowel assumptions, this would make us overly wide. Using less than 8 times thickness for a tensile member would risk interfering with the plywood backboard. It is for these reasons that we opted for single dowel pins.

We now looked at optimizing the pins of the system to not have dead dowel weight. We did this by trying to minimize pin bending moment and approach to shear failure load of the pins. To do this we measure the inside and outside diameters of the 3 hooks (not knowing that we could move the pins) and optimized for our best case, which was the closest hooks (as the worse case was not much different).

We used a wide tensile member design to achieve the maximum shear at both the support and load pins. This was done by having the tensile members hug the hooks and the hanger which was allowed to have 7mm of room (6.35mm diameter steel rod for hanger hook). As there was no more room on the outside of the tensile members to support box beams of adequate moment of inertia, we decided to fork our tensile members and have the box beams inside of the tensile members. The moment of inertia of the solid cross section was insufficient so we went with a stepped design and assumed that the connection would be relatively rigid (acts as two separate box beams with one rigid support and one fixed support). The box beam effective lengths were modeled as 2L. These calculations and optimizations for the tensile members and compressive members can be seen below in Figure 4.2 and Figure 4.3.

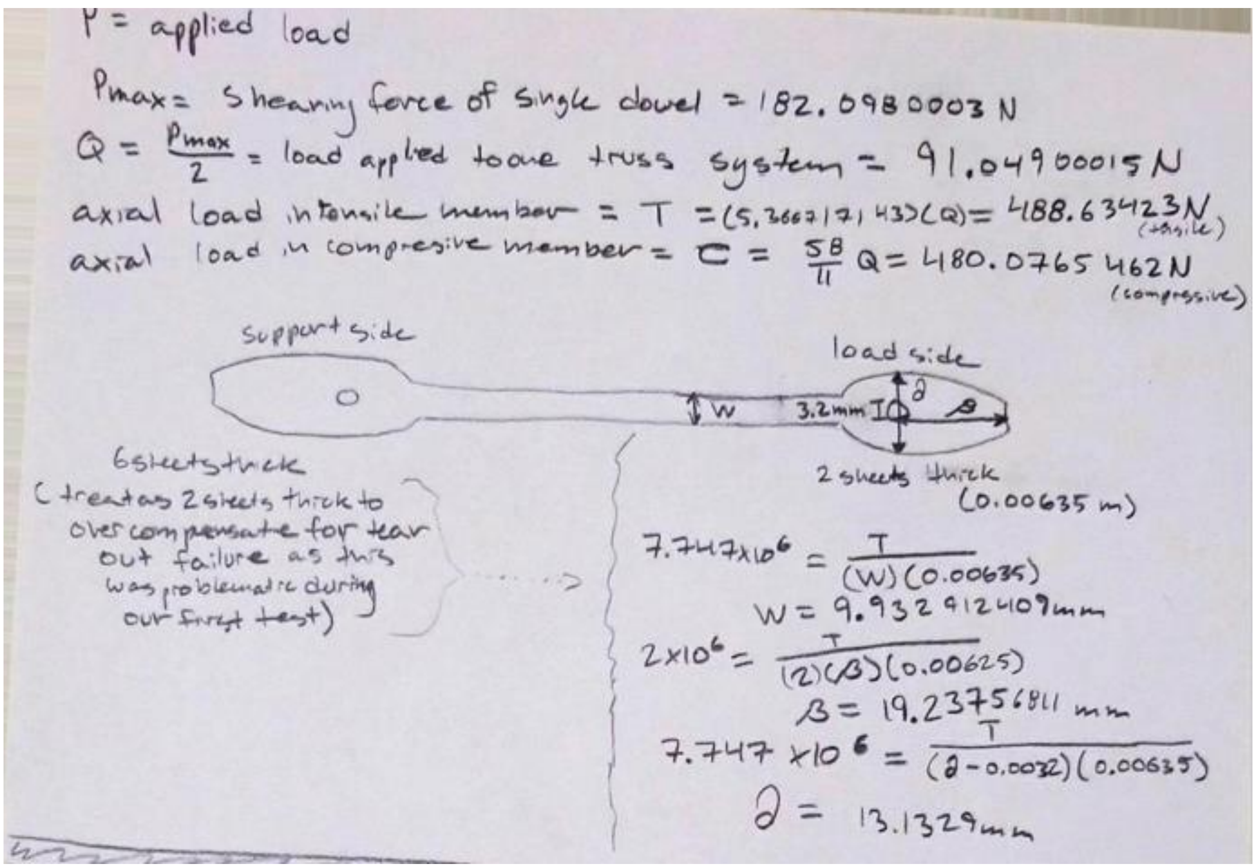


Figure 4.2 - Optimization of Tensile Members

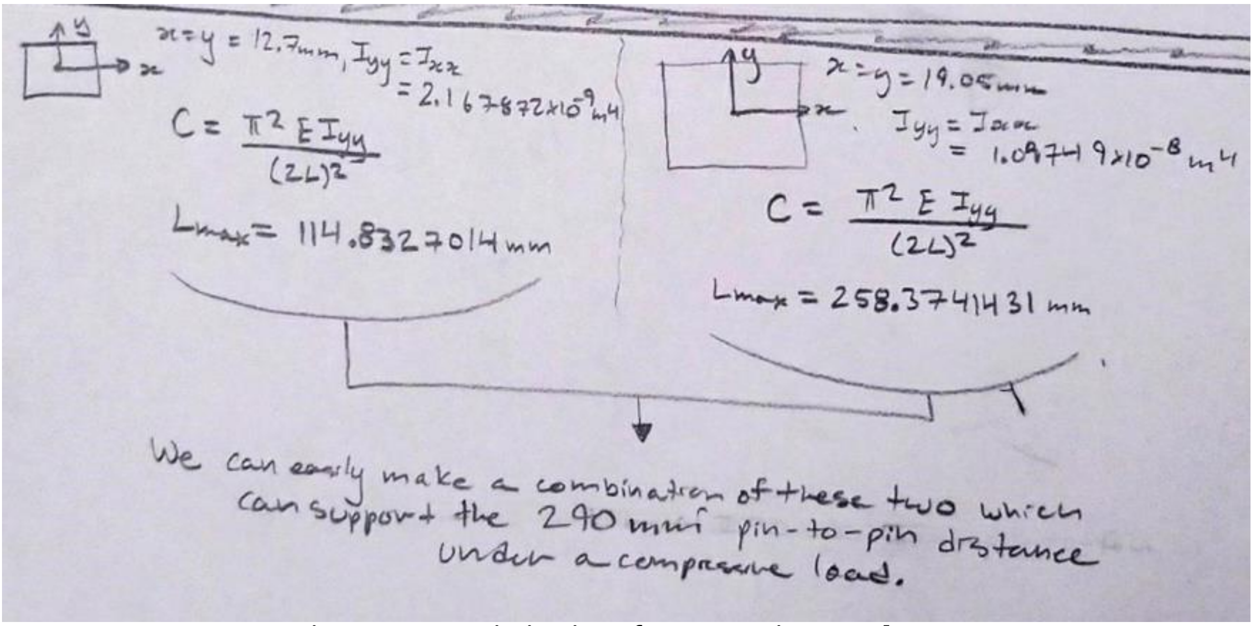


Figure 4.3 - Optimization of Compressive Members

The above calculations were optimized by trying to achieve equal probability of failure in all members when the max load (max shear of the pins) is reached. Not all modes of failure turned out equal. Over compensation for tear out at the hooks was done (as we failed to this mode in our trial run) in the tensile members and as mentioned earlier, the effective lengths of the box beam were overkill. Also, the zero force members were intentionally made a little larger than necessary as they become loaded as deformation occurs. The most likely modes of failure are tear out at the load pin, tensile failure in the middle of the member, tensile failure around the load and support pins, and shearing of the load pin, as these were all optimized to have equal probability of failure. It is possible to fail due to bending moment but is not of great concern as load pin fails to shear before failing due to bending moments as seen bellow.

5 CONSTRUCTION & TEST RESULTS

5.1 Design Obstacles

The main design obstacle encountered was where the double box beam designs were too large to fit in the testing rig. The main culprits where top and front pins which ere made of three close packed dowels, as well as the double box beams placed on the outside of each of the tensile members. These designs were around 9-cm wide, versus the 6-cm wide test rig, as shown earlier in Figure 3.3. The box beams served the purpose of distributing the compressive load over a larger area, as well as helping stabilize the outer edges. The size could not be easily compressed as this would affect the failure points of the design. To solve this, the box beams were put together and redesigned to be placed inline with the tensile members. This design is shown below in Figure 5.1.

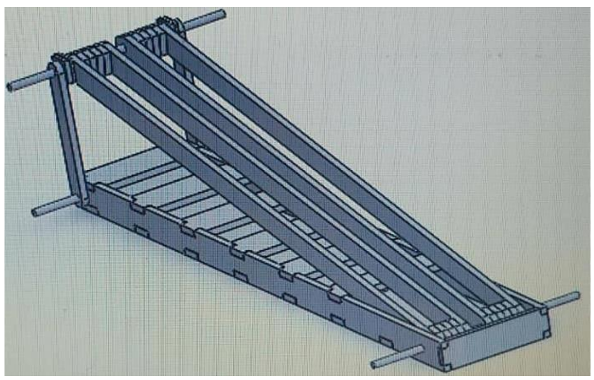


Figure 5.1 - Compressed Box Beam Design

This approach allowed the design to be significantly compressed, allowing it to fit inside the testing rig. However, this caused a legality problem as the truss needed to be of adjustable width. This was solved by splitting the box beams again, but keeping them behind the tensile as before. This design was shown earlier in Figure 3.4.

5.2 Design Optimization

This crane was designed based on the concept of a larger, stronger structure being less likely to allow localized material deficiencies to have effect on the integrity of the crane. The pins' shear strength was used to determine the maximum design load. With recognition that additional pins introduce construction complexity and weight, a three-pin design was determined to optimally recognize the goals.

As tensile members can achieve any strength requirements, simply by increasing their cross-sectional area, the compressive member required the bulk of optimization efforts. A box beam was selected, both for its easy construction and relatively high second moment of inertia for a given cross-sectional area. The beam was designed to exceed the maximum possible force before shearing of the pins, while remaining clear of the tensile members. The tensile members used widened bases to reduce bending in the pin, and place most of the load in shear.

Further study resulted in the box beams required being too large to conform to the design constraints. A solid beam was used instead, maintaining the same philosophy, but adding strength, though less efficiently.

Finally, the tensile members were thickened at the joints in width and height, to reduce tearout and shear failure at the joints. This was also optimized to the yield strength of the tensile members, to avoid overdesign in either dimension.

5.3 Initial Test Results

Initial testing indicated the solid beams were able to effectively satisfy their design requirements. The tensile members, however, sheared prematurely at the joints, indicating potential miscalculation in this area. It also gave a PV of approx. 40, not nearly within an acceptable value. Further analysis of the structure indicated the tensile members failed prematurely due to fractures surrounding the pin. It was believed that the structural integrity of the balsa may have been compromised in construction, and the piece fractured due to this. Additional reinforcement was added to the ends to ensure this would no longer pose an issue.

6 CONCLUSIONS & RECOMMENDATIONS

Through the design and construction of a cantilevered truss, the theoretical concepts we have learnt through several of our classes were put to the test in real-life scenarios. The similarities and differences between designing a truss that would work theoretically and one that would perform well in reality were an important discovery for the team members.

Through multiple iterations, the group attempted to optimize the design and increase the PV of the structure. For the reasons described above, the truss of choice had a simple design—a triangle, with stacked tensile members and solid box beams for its compressive members—and a strength to mass ratio of 502.69.

The initial tests did not go as well as expected and yielded a PV that was lower than the one calculated theoretically, mainly given to inaccuracies during the build process of the truss. Modifications were made to the design, and so it is expected to perform well on demo-day.

There are further design improvements that the group would have wanted to make to the truss, yet were unable to due to time constraints. Namely, making lighter box beams—given that these were over-engineered, and are comparatively too heavy and unnecessarily strong for the purposes of this truss. The team would have liked to make hollow box beams, with a slightly larger perimeter than the ones that are currently being used; however, time constraints and limitations in the amount of material we had left made this unachievable. Additionally, further optimizing the shape and size of the tensile members would have been ideal, as is would have been optimal to have the tensile and compressive members break at theoretically the same time—to avoid having any unnecessary mass.

7 REFERENCES

1. Beer, Ferdinand P., et al. *Mechanics of Materials.* 7th ed., McGraw-Hill Education Private Limited, 2017.